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Multi-terminal superconducting phase qubit

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Abstract

Mesoscopic multi-terminal Josephson junctions are novel devices that provide weak coupling between several bulk superconductors through a common normal layer. Because of the nonlocal coupling of the superconducting banks, a current flow between two of the terminals can induce a phase difference and/or current flow in the other terminals. This "phase dragging" effect is used in designing a new type of superconducting phase qubit, the basic element of a quantum computer. Time-reversal symmetry breaking can be achieved by inserting a π-phase shifter into the flux loop. Logical operations are done by applying currents. This removes the necessity for local external magnetic fields to achieve bistability or controllable operations.

Keywords: Quantum computing; Qubit; Multi-terminal; Josephson junction

Although time-domain coherent oscillations have been observed in superconducting charge qubits [1], the short decoherence time τ_φ, due to the fluctuations of the background charges, prevents these qubits from being good candidates for large-scale quantum computing. Phase qubits, on the other hand, can couple weakly to the background charges and therefore potentially have larger τ_φ. To achieve a reasonably long τ_φ, it is necessary to have a "quiet" [2] phase qubit—with small magnetic coupling to the environment, or equivalently, small inductance. A usual rf-SQUID can show bistability only when the inductance L of the ring exceeds $2\pi\Phi_0/I_c$ [3], and therefore cannot be quiet. Here, I_c is the Josephson critical current of the junction and $\Phi_0 = h/2e$ the flux quantum. To overcome this problem, three Josephson junctions have been included in a superconducting ring [4]. One of the three Josephson phases is fixed by the other two and the external flux, which leaves the SQUID with two degrees of freedom, making bistability possible even when $L = 0$.

A four-terminal junction is described by three phase variables (see below). Connecting two of the terminals by a superconducting ring will fix one of the phases to the external flux (neglecting the inductance of the loop, L). The resulting four-terminal SQUID [5] will have two degrees of

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freedom and can exhibit bistability at small L. Bistability of a four-terminal SQUID made from microbridges has been observed experimentally [6]. As we shall see, with a mesoscopic four-terminal junction (Fig. 1a) it is possible to have bistability even at $L = 0$, due to the phase-dragging effect [7,8].

A mesoscopic four-terminal junction is shown in Fig. 1a. The four bulk superconductors are connected to each other via a two-dimensional electron gas (2DEG) region. The phase of the order parameter in the ith terminal is denoted by ϕ_i. When the dimensions of the 2DEG region are smaller than the superconducting coherence length in the banks, the total current I_i flowing into the ith terminal depends on the superconducting phases ϕ_j in all the banks through [7]

$$I_i = \frac{\pi \Delta_0}{e} \sum_{j=1}^{4} \gamma_{ij} \sin\frac{\phi_{ij}}{2} \tanh\left[\frac{\Delta_0}{2T}\cos\frac{\phi_{ij}}{2}\right], \tag{1}$$

where γ_{ij} are Josephson coupling constants,[1] $\phi_{ij} \equiv \phi_i - \phi_j$, and Δ_0 is the superconducting gap. We study the system at temperatures close to $T = 0$, where decoherence due to the environment is minimal. In this limit, the Josephson energy associated with the four-terminal junction is given by

$$\mathscr{E}_J \equiv \frac{E_J}{E_0} = -\frac{1}{\gamma_{12}} \sum_{i<j} \gamma_{ij} \left|\cos\frac{\phi_{ij}}{2}\right|. \tag{2}$$

Here, $E_0 = \hbar I_0/e$ and $I_0 = \pi\gamma_{12}\Delta_0/e$ are the Josephson energy and critical current for the subjunction 1–2 at $T = 0$, respectively.

A mesoscopic four-terminal SQUID is constructed from the four-terminal junction by connecting two of the terminals via a superconducting ring (Fig. 1b, ignore the π-junction for the moment). We label the terminals in such a way that subjunction 1–2 forms the bias circuit carrying current I, and subjunction 3–4 makes the flux loop with current J and flux Φ threading the ring. We introduce new variables by $\phi_{1,2} = (\mp\theta + \chi)/2$ and $\phi_{3,4} = (\pm\phi - \chi)/2$, implicitly setting $\sum \phi_i = 0$, which is allowed because the overall phase is arbitrary. The phase differences θ and ϕ are between terminals 1–2 and 3–4, respectively. On the other hand, χ is the overall phase difference between the ring and the bias circuit. It is also useful to define the new dimensionless parameters

$$\begin{aligned}
\gamma &= (\gamma_{13} + \gamma_{23} + \gamma_{14} + \gamma_{24})/\gamma_{12}, \\
\epsilon &= (\gamma_{13} + \gamma_{23} - \gamma_{14} - \gamma_{24})/\gamma_{12}, \\
\delta_1 &= (\gamma_{13} - \gamma_{23} - \gamma_{14} + \gamma_{24})/\gamma_{12}, \\
\delta_2 &= (\gamma_{13} - \gamma_{23} + \gamma_{14} - \gamma_{24})/\gamma_{12}, \\
\kappa &= \gamma_{34}/\gamma_{12}.
\end{aligned} \tag{3}$$

In general our system has a 3D phase space (ϕ, θ, χ). However, we are interested in the regime where $\kappa \ll \gamma \ll 1$ and $L \to 0$, so that the self-generated flux by the ring be very small ($\propto \gamma I_0 L \ll \Phi_0$). Therefore, ϕ is practically fixed by the external field and/or by a π-phase shifter inserted into the ring (see below), and we can study the system in the 2D phase space of (θ, χ).

Applying an external flux $\Phi_e = \Phi_0/2$ to the superconducting ring makes the system bistable (as in the rf-SQUID or three-junction cases), meaning that the free energy of the system has two local minima, corresponding to opposite directions of current in the ring. Note that the external flux is not used to manipulate the flux (qubit) state. It therefore can be fixed to $\Phi_0/2$ for all qubits. This opens the possibility of replacing the external fluxes by a π-phase shifter [9] in each qubit's superconducting ring, as shown in Fig. 1b. The net effect is the same but this has the advantage that the π-phase shifter does not bring in extra coupling to the electromagnetic environment. In the regime of interest ($L \to 0$), $\phi = \pi$ and the free energy of the system is given by

$$\mathscr{U} = -\mathscr{I}\theta + \mathscr{E}_J(\theta, \chi, \phi = \pi), \tag{4}$$

where $\mathscr{I} \equiv I/I_0$. Contour plots of this free energy at two different sets of parameters are given in Figs. 2 and 3a. Tunneling between the potential wells is enabled by charging effects, with the electrostatic capacitance of the system defining the effective "mass" in the kinetic energy term (see e.g., Ref. [10, Section 2.2.2]).

When $\mathscr{I} = 0$, the two minima of $\mathscr{U}$ have equal energy. Contour plots of the free energy of the system at $\mathscr{I} = 0$ are shown in Fig. 2. As is clear

[1] Analytical expressions for γ_{ij} in the case of a clean rectangular 2DEG region are given in Ref. [7].

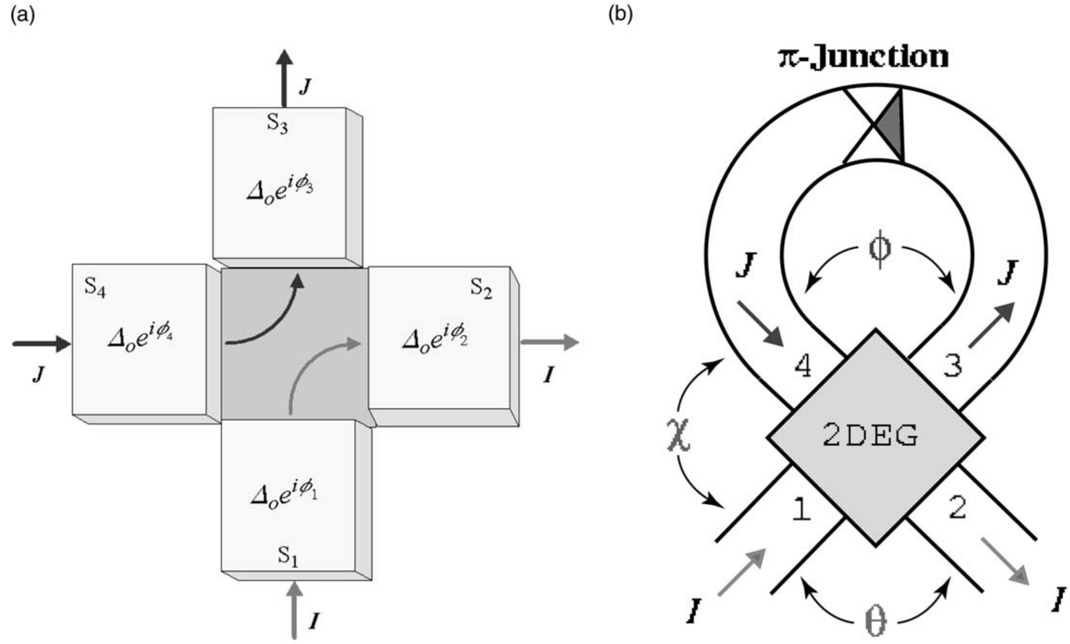

Fig. 1. Mesoscopic (a) four-terminal junction and (b) four-terminal SQUID. The π-junction is included in the ring to attain bistability.

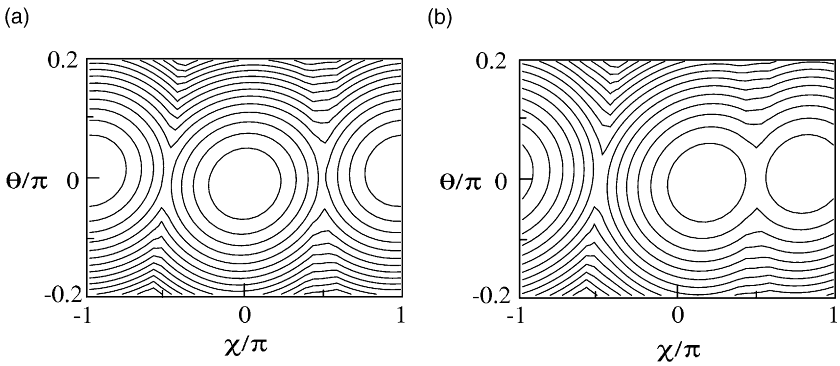

Fig. 2. Contour plot of the free energy at $T = 0$ and $\mathscr{I} = 0$. Junction with (a) $\epsilon = 0$, (b) $\epsilon = 0.03$. Other parameters are $\gamma = 0.1$, $\delta_1 = \delta_2 = 0.05$.

from the figure, with the parameters chosen, the minima are located very close to $\theta = 0$. In extended phase space, there are also other minima, near $\theta = 2\pi n$ (n an integer). Those minima are separated from the ones shown in the figure by high and wide potential barriers. Therefore, tunneling in those directions is negligible. The situation is different for χ. When $\epsilon = 0$, as is the case for a system with a square 2DEG region and four equivalent terminals, the minima are equidistant at $\chi = 0, \pm\pi$, with equal barriers between them (Fig. 2a). Therefore the tunneling probabilities in the left and right directions are the same. This is undesirable for qubit application because it makes the system sensitive to random charges in the environment [11,12]. However, making $\epsilon \neq 0$ will move two of the minima closer together, making the barrier heights unequal (Fig. 2b). Pairs of

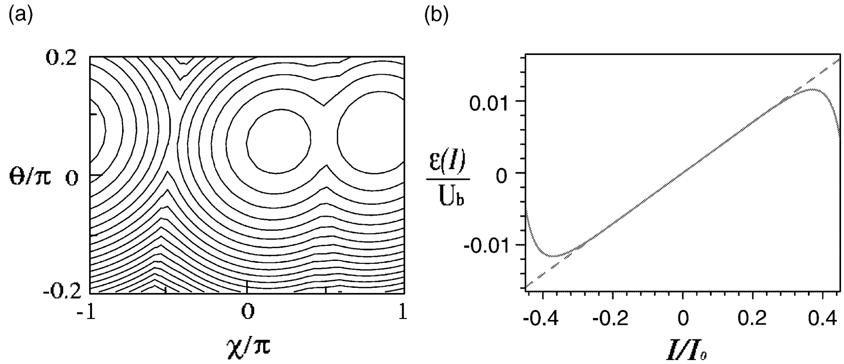

Fig. 3. (a) Contour plot of the free energy for the system of Fig. 2b at $\mathscr{I} = 0.05$. (b) Solid line is the energy bias as a function of the transport current, normalized to the barrier height, for the system of Fig. 2b. Dashed line is the linear approximation.

minima are then isolated, and one can associate a given pair of minima with the logical qubit states $\{|0\rangle, |1\rangle\}$. This regime can be achieved easily by choosing a rectangular 2DEG region instead of a square one [13].

Applying a nonzero transport current $\mathscr{I}$ moves the minima from being centered around $\theta = 0$ to some $\theta = \theta_0(\mathscr{I})$. More importantly, it removes their degeneracy. Fig. 3a displays the contour plot for $\mathscr{U}$ using the parameters of Fig. 2b, but with $\mathscr{I} = 0.05$. As a result of the applied current, the two minima are now clearly unequal.

The energy difference between the two minima $\varepsilon(\mathscr{I})$ is plotted in Fig. 3b. As is evident from the figure, this energy bias is linearly dependent on $\mathscr{I}$ for a relatively wide range of the transport current: $-0.3 \lesssim \mathscr{I} \lesssim 0.3$. We can therefore approximate it by $\varepsilon(\mathscr{I}) = \varepsilon_0\mathscr{I}$, where ε_0 is given by [13]

$$\varepsilon_0 = \frac{(\gamma\delta_1 - \epsilon\delta_2)[\gamma\epsilon(\delta_1^2 + \delta_2^2) + \delta_1\delta_2(\gamma^2 + \epsilon^2)]}{4(\gamma^2 + \epsilon^2)}. \tag{5}$$

To study the quantum dynamics of this system, we need to know the capacitances between the terminals of the four-terminal junction. In general, there exists a capacitance between any two terminals of the system and one has to find the component of the effective mass tensor along the direction of tunneling in the same way as in Ref. [11]. However, as is clear from Fig. 2b, the difference in θ (and also ϕ) from one minimum to another is very small compared to that in χ. The tunneling is therefore effectively in the χ direction. Using a simplified 1D model we find the tunneling matrix element at $\mathscr{I} = 0$ to be $\Delta \sim \hbar\omega_0 \times \exp(-(U_b/E_c)^{-1/2})$, where $\omega_0 = [8(\gamma^2 + \epsilon^2)]^{1/4} \times (E_0E_c)^{1/2}$ is the plasma frequency at the minima, $E_c = e^2/2C_{\text{eff}}$ is the charging energy, C_{eff} is the effective capacitance in the direction of tunneling, and

$$U_b = \frac{(\gamma - |\epsilon|)^2 E_0}{2\left[\gamma + |\epsilon| + \sqrt{2(\gamma^2 + \epsilon^2)}\right]} \tag{6}$$

is the barrier height between the two nearest minima [13].

As mentioned above, an $\mathscr{I} \neq 0$ lifts the degeneracy between the lowest-energy states and therefore stops the coherent tunneling. This energy difference induces a relative phase between the logical states. Therefore, control over the transport current suffices to manipulate the effective one-qubit Hamiltonian $H_{\text{eff}} = \Delta(I)\sigma_x + E_0\varepsilon(I)\sigma_z$. Entangling operations between two qubits are possible through voltage-controlled couplings provided by additional 2DEGs [13]. Combining the two regimes of zero and nonzero $\mathscr{I}$ and using two-qubit coupling, it is possible to perform any quantum gate operations [14].

Most of the arguments about decoherence discussed in Refs. [3,15] carry over to this system. There are however also two sources of decoherence different from those discussed in the references. The first is decoherence caused by fluctuations of the transport current $\mathscr{I}$. This can be reduced by increasing the internal resistance of the current source and working at low temperatures [3,13]. Moreover, the current carried by the quasiparticles through the normal region can also cause decoherence. As shown in Ref. [16], the quasiparticle (shunt) resistance is $R_{qp} \propto T\cosh^2(E_A/2k_BT)$, where $\pm E_A$ are the energies of the Andreev bound states inside the normal region. To achieve (exponentially) large R_{qp} and therefore long τ_φ, it is necessary to work at temperatures far below E_A. In systems with a large 2DEG region, the energy scale E_A is inversely proportional to the dimensions of the normal region and can be much smaller than the gap Δ_0 (which determines the energy scale in tunnel junctions). For short junctions on the other hand (which is the case here), $E_A \sim \Delta_0\cos(\Delta\phi/2)$ and can be large if the phase difference $\Delta\phi$ is not close to $\pi/2$. A phase-dependent conductance in agreement with the above picture has been observed [17].

In the limit studied in this paper, the time scale of the dynamics is set by the Josephson and charging energies, as well as by the coupling coefficients (3). For a junction size of 100 nm, we estimate $I_0 \sim 10^{-7}$ A [18] and $C_{\text{eff}} \sim 10^{-13}$ F. Taking $\gamma = 0.1$, $\delta_1 = \delta_2 = 0.05$ and $\varepsilon = 0.04$, we obtain $\Delta \sim 0.1$ GHz while tunneling through the barrier separating the pairs of minima is 10^{-3} smaller. The dynamics is thus effectively restricted to one pair of minima in phase space. Moreover, from Eq. (5) we obtain $E_0\varepsilon_0 \sim 0.01$ GHz. Using the latter result, we estimate that up to 10^5 operations can be performed within the decoherence time due to fluctuations of the transport current [13].

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